MAA

MATHEMATICAL ASSOCIATION OF AMERICA

MAA American Mathematics Competitions

32nd Annual

AMC 8

American Mathematics Contest 8

Tuesday, November 15, 2016

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record your information on the answer form.
8. You will have 40 minutes to complete the test once your proctor tells you to begin.
9. When you finish the exam, *sign your name* in the space provided on the answer form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules.

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2016

AMC 8

DO NOT OPEN UNTIL TUESDAY, NOVEMBER 15, 2016

****ADMINISTRATION ON AN EARLIER DATE WILL DISQUALIFY YOUR SCHOOL'S RESULTS****

1. PLEASE READ THE TEACHERS' MANUAL BEFORE NOVEMBER 15, 2016. All rules and instructions needed to administer this exam are contained in the manual. You will not need anything from inside this package until November 15.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be sent by trackable mail to the AMC office no later than 24 hours following the exam.
4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.
5. The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules.

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1. The longest professional tennis match ever played lasted a total of 11 hours and 5 minutes. How many minutes was this?

   (A) 605 **(B)** 655 **(C)** 665 **(D)** 1005 **(E)** 1105

2. In rectangle $ABCD$, $AB = 6$ and $AD = 8$. Point M is the midpoint of $\overline{AD}$. What is the area of $\triangle AMC$?

   (A) 12 **(B)** 15 **(C)** 18 **(D)** 20 **(E)** 24

3. Four students take an exam. Three of their scores are 70, 80, and 90. If the average of their four scores is 70, then what is the remaining score?

   (A) 40 **(B)** 50 **(C)** 55 **(D)** 60 **(E)** 70

4. When Cheenu was a boy he could run 15 miles in 3 hours and 30 minutes. As an old man he can now walk 10 miles in 4 hours. How many minutes longer does it take for him to walk a mile now compared to when he was a boy?

   (A) 6 **(B)** 10 **(C)** 15 **(D)** 18 **(E)** 30

5. The number N is a two-digit number.

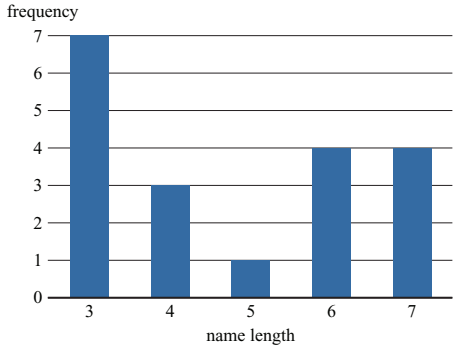
   - When N is divided by 9, the remainder is 1.
   - When N is divided by 10, the remainder is 3.

   What is the remainder when N is divided by 11?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 5 **(E)** 7

6. The following bar graph represents the length (in letters) of the names of 19 people. What is the median length of these names?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7



7. Which of the following numbers is **not** a perfect square?

(A) 1^{2016} **(B)** 2^{2017} **(C)** 3^{2018} **(D)** 4^{2019} **(E)** 5^{2020}

8. Find the value of the expression

$$100-98+96-94+92-90+\cdots+8-6+4-2.$$

(A) 20 **(B)** 40 **(C)** 50 **(D)** 80 **(E)** 100

9. What is the sum of the distinct prime integer divisors of 2016?

(A) 9 **(B)** 12 **(C)** 16 **(D)** 49 **(E)** 63

10. Suppose that $a*b$ means $3a-b$. What is the value of x if

$$2*(5*x)=1\ ?$$

(A) $\frac{1}{10}$ **(B)** 2 **(C)** $\frac{10}{3}$ **(D)** 10 **(E)** 14

11. Determine how many two-digit numbers satisfy the following property: When the number is added to the number obtained by reversing its digits, the sum is 132.

(A) 5 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 12

12. Jefferson Middle School has the same number of boys and girls. Three-fourths of the girls and two-thirds of the boys went on a field trip. What fraction of the students on the field trip were girls?

(A) $\frac{1}{2}$ **(B)** $\frac{9}{17}$ **(C)** $\frac{7}{13}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{14}{15}$

13. Two different numbers are randomly selected from the set $\{-2,-1,0,3,4,5\}$ and multiplied together. What is the probability that the product is 0?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

14. Karl's car uses a gallon of gas every 35 miles, and his gas tank holds 14 gallons when it is full. One day Karl started with a full tank of gas, drove 350 miles, bought 8 gallons of gas, and continued driving to his destination. When he arrived, his gas tank was half full. How many miles did Karl drive that day?

(A) 525 **(B)** 560 **(C)** 595 **(D)** 665 **(E)** 735

15. What is the largest power of 2 that is a divisor of $13^4 - 11^4$?

(A) 8 **(B)** 16 **(C)** 32 **(D)** 64 **(E)** 128

16. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead, because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

(A) $1\frac{1}{4}$ **(B)** $3\frac{1}{3}$ **(C)** 4 **(D)** 5 **(E)** 25

17. An ATM password at Fred's Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence 9, 1, 1, then how many passwords are possible?

(A) 30 **(B)** 7290 **(C)** 9000 **(D)** 9990 **(E)** 9999

18. In an All-Area track meet, 216 sprinters enter a 100-meter dash competition. The track has 6 lanes, so only 6 sprinters can compete at a time. At the end of each race the five non-winners are eliminated, and the winner will compete again in a later race. How many races are needed to determine the champion sprinter?

(A) 36 **(B)** 42 **(C)** 43 **(D)** 60 **(E)** 72

19. The sum of 25 consecutive even integers is $10,000$. What is the largest of these 25 consecutive even integers?

(A) 360 **(B)** 388 **(C)** 412 **(D)** 416 **(E)** 424

20. The least common multiple of a and b is 12, and the least common multiple of b and c is 15. What is the least possible value of the least common multiple of a and c?

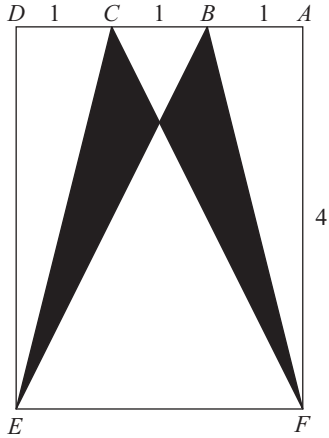
(A) 20 **(B)** 30 **(C)** 60 **(D)** 120 **(E)** 180

21. A box contains 3 red chips and 2 green chips. Chips are drawn randomly, one at a time without replacement, until all 3 of the reds are drawn or until both green chips are drawn. What is the probability that the 3 reds are drawn?

(A) $\frac{3}{10}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{2}{3}$

22. Rectangle $DEFA$ below is a 3×4 rectangle with $DC = CB = BA = 1$. The area of the "bat wings" (the shaded area) is

(A) 2 **(B)** $2\frac{1}{2}$ **(C)** 3 **(D)** $3\frac{1}{2}$ **(E)** 4



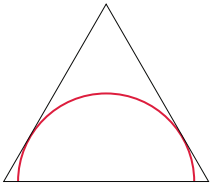
23. Two congruent circles centered at points A and B each pass through the other's center. The line containing both A and B is extended to intersect the circles at points C and D. The two circles intersect at two points, one of which is E. What is the degree measure of $\angle CED$?

(A) 90 **(B)** 105 **(C)** 120 **(D)** 135 **(E)** 150

24. The digits 1, 2, 3, 4, and 5 are each used once to write a five-digit number $PQRST$. The three-digit number PQR is divisible by 4, the three-digit number QRS is divisible by 5, and the three-digit number RST is divisible by 3. What is P?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

25. A semicircle is inscribed in an isosceles triangle with base 16 and height 15 so that the diameter of the semicircle is contained in the base of the triangle as shown. What is the radius of the semicircle?



(A) $4\sqrt{3}$ **(B)** $\frac{120}{17}$ **(C)** 10 **(D)** $\frac{17\sqrt{2}}{2}$ **(E)** $\frac{17\sqrt{3}}{2}$

SOLUTIONS

Your School Manager will be sent at least one copy of the 2016 AMC 8 Solutions Pamphlet with the report. It is meant to be loaned to students (but not duplicated).

WRITE TO US

Comments about the problems and solutions for this AMC 8 should be addressed to:

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934 Nicolet Ave
Oshkosh, WI 54901-1634

Comments about administrative arrangements should be addressed to:
MAA American Mathematics Competitions / amcinfo@maa.org

MAA American Mathematics Competitions
PO Box 471
Annapolis Junction, MD 20701

AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute, multiple choice contests. All schools participating in the AMC 8 receive a brochure and registration form for the 2015 AMC 10. Schools with high scoring students on the AMC 8 should consider administering the AMC 10. The best way to prepare for these contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

MAA American Mathematics Competitions
PO Box 471
Annapolis Junction, MD 20701

PUBLICATIONS

A complete listing of the current publications for sale can be found on our web site:
maa.org/math-competitions